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A Software Simulation Study of a (255,223) Reed-Solomon Encoder/Decoder

Fabrizio Pollara

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ABSTRACT

A set of software programs which simulates a (255,223) Reed-Solomon encoder/decoder pair is described. The transform decoder algorithm uses a modified Euclid algorithm, and closely follows the pipeline architecture proposed for the hardware decoder. Uncorrectable error patterns are detected by a simple test, and the inverse transform is computed by a finite field FFT.

Numerical examples of the decoder operation are given for some test codewords, with and without errors. The use of the software package is briefly described.

1. INTRODUCTION

A (255,223) Reed-Solomon (RS) code has been adopted as the standard outer code for concatenated coding systems by NASA and by the European Space Agency (ESA) [1]. This particular RS code is defined in $GF(2^8)$ by the following parameters:

$N = 255$ = number of 8-bit symbols in a codeword (block)

$K = 223$ = number of information symbols in a block

$T = N-K$ = number of parity symbols.

Such a code is capable of correcting up to $T/2 = 16$ symbol errors in a block. The generator polynomial $g(x)$ of the code is given by,

$$g(x) = \prod_{i=M}^{M+T+1} (x - \alpha^{Gi}) = \sum_{j=0}^T g_j x^j \quad (1)$$

where $M = 112$, $G = 11$, and α is a root of the primitive polynomial over $GF(2)$

$$x^8 + x^7 + x^2 + x + 1$$

Every element of $GF(2^8)$ can be represented as a polynomial in α over $GF(2)$ of degree less than 8, as shown in Table 1.

The constant M is chosen so that the polynomial has symmetrical coefficients, i.e.,

$$g_j = g_{T-j}, \quad j=0,1,\dots,T$$

It is shown in [2] that this is true if $M = 2^{8-1} - (T/2) = 112$.

The constant $G = 11$ is chosen to minimize the bit-serial implementation complexity of the encoder [3]. The polynomial coefficients are shown in Table 2.

Table 1. Decimal Representation of Elements of $GF(2^8)$

$$z = \alpha^x; x = \alpha^y$$

| x | y | z | x | y | z | x | y | z | x | y | z |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0 | 0 | 1 | 1 | 0 | 2 | 2 | 1 | 4 | 3 | 99 | 8 |
| 4 | 2 | 16 | 5 | 198 | 32 | 6 | 100 | 64 | 7 | 106 | 128 |
| 8 | 3 | 135 | 9 | 205 | 137 | 10 | 199 | 149 | 11 | 188 | 173 |
| 12 | 101 | 221 | 13 | 126 | 61 | 14 | 107 | 122 | 15 | 42 | 244 |
| 16 | 4 | 111 | 17 | 141 | 222 | 18 | 206 | 59 | 19 | 78 | 118 |
| 20 | 200 | 236 | 21 | 212 | 95 | 22 | 189 | 190 | 23 | 225 | 251 |
| 24 | 102 | 113 | 25 | 221 | 226 | 26 | 127 | 67 | 27 | 49 | 134 |
| 28 | 108 | 139 | 29 | 32 | 145 | 30 | 43 | 165 | 31 | 243 | 205 |
| 32 | 5 | 29 | 33 | 87 | 58 | 34 | 142 | 116 | 35 | 232 | 232 |
| 36 | 207 | 87 | 37 | 172 | 174 | 38 | 79 | 219 | 39 | 131 | 49 |
| 40 | 201 | 98 | 41 | 217 | 196 | 42 | 213 | 15 | 43 | 65 | 30 |
| 44 | 190 | 60 | 45 | 148 | 120 | 46 | 226 | 240 | 47 | 180 | 103 |
| 48 | 103 | 206 | 49 | 39 | 27 | 50 | 222 | 54 | 51 | 240 | 108 |
| 52 | 128 | 216 | 53 | 177 | 55 | 54 | 50 | 110 | 55 | 53 | 220 |
| 56 | 109 | 63 | 57 | 69 | 126 | 58 | 33 | 252 | 59 | 18 | 127 |
| 60 | 44 | 254 | 61 | 13 | 123 | 62 | 244 | 246 | 63 | 56 | 107 |
| 64 | 6 | 214 | 65 | 155 | 43 | 66 | 88 | 86 | 67 | 26 | 172 |
| 68 | 143 | 223 | 69 | 121 | 57 | 70 | 233 | 114 | 71 | 112 | 228 |
| 72 | 208 | 79 | 73 | 194 | 158 | 74 | 173 | 187 | 75 | 168 | 241 |
| 76 | 80 | 101 | 77 | 117 | 202 | 78 | 132 | 19 | 79 | 72 | 38 |
| 80 | 202 | 76 | 81 | 252 | 152 | 82 | 218 | 183 | 83 | 138 | 233 |
| 84 | 214 | 85 | 85 | 84 | 170 | 86 | 66 | 211 | 87 | 36 | 33 |
| 88 | 191 | 66 | 89 | 152 | 132 | 90 | 149 | 143 | 91 | 249 | 153 |
| 92 | 227 | 181 | 93 | 94 | 237 | 94 | 181 | 93 | 95 | 21 | 186 |
| 96 | 104 | 243 | 97 | 97 | 97 | 98 | 40 | 194 | 99 | 186 | 3 |
| 100 | 223 | 6 | 101 | 76 | 12 | 102 | 241 | 24 | 103 | 47 | 48 |
| 104 | 129 | 96 | 105 | 230 | 192 | 106 | 178 | 7 | 107 | 63 | 14 |
| 108 | 51 | 28 | 109 | 238 | 56 | 110 | 54 | 112 | 111 | 16 | 224 |
| 112 | 110 | 71 | 113 | 24 | 142 | 114 | 70 | 155 | 115 | 166 | 177 |
| 116 | 34 | 229 | 117 | 136 | 77 | 118 | 19 | 154 | 119 | 247 | 179 |
| 120 | 45 | 225 | 121 | 184 | 69 | 122 | 14 | 138 | 123 | 61 | 147 |
| 124 | 245 | 161 | 125 | 164 | 197 | 126 | 57 | 13 | 127 | 59 | 26 |
| 128 | 7 | 52 | 129 | 158 | 104 | 130 | 156 | 208 | 131 | 157 | 39 |
| 132 | 89 | 78 | 133 | 159 | 156 | 134 | 27 | 191 | 135 | 8 | 249 |
| 136 | 144 | 117 | 137 | 9 | 234 | 138 | 122 | 83 | 139 | 28 | 166 |
| 140 | 234 | 203 | 141 | 160 | 17 | 142 | 113 | 34 | 143 | 90 | 68 |
| 144 | 209 | 136 | 145 | 29 | 151 | 146 | 195 | 169 | 147 | 123 | 213 |
| 148 | 174 | 45 | 149 | 10 | 90 | 150 | 169 | 180 | 151 | 145 | 239 |
| 152 | 81 | 39 | 153 | 91 | 178 | 154 | 118 | 227 | 155 | 114 | 65 |
| 156 | 133 | 130 | 157 | 161 | 131 | 158 | 73 | 129 | 159 | 235 | 133 |
| 160 | 203 | 141 | 161 | 124 | 157 | 162 | 253 | 189 | 163 | 196 | 253 |
| 164 | 219 | 125 | 165 | 30 | 250 | 166 | 139 | 115 | 167 | 210 | 230 |
| 168 | 215 | 75 | 169 | 146 | 150 | 170 | 85 | 171 | 171 | 170 | 209 |
| 172 | 67 | 37 | 173 | 11 | 74 | 174 | 37 | 148 | 175 | 175 | 175 |
| 176 | 192 | 217 | 177 | 115 | 53 | 178 | 153 | 106 | 179 | 119 | 212 |
| 180 | 150 | 47 | 181 | 92 | 94 | 182 | 250 | 188 | 183 | 82 | 255 |
| 184 | 228 | 21 | 185 | 236 | 242 | 186 | 95 | 99 | 187 | 74 | 198 |
| 188 | 182 | 11 | 189 | 162 | 22 | 190 | 22 | 44 | 191 | 134 | 88 |
| 192 | 105 | 176 | 193 | 197 | 231 | 194 | 98 | 73 | 195 | 254 | 146 |
| 196 | 41 | 163 | 197 | 125 | 193 | 198 | 187 | 5 | 199 | 204 | 10 |
| 200 | 224 | 20 | 201 | 211 | 40 | 202 | 77 | 80 | 203 | 140 | 160 |
| 204 | 242 | 199 | 205 | 31 | 9 | 206 | 48 | 18 | 207 | 220 | 36 |
| 208 | 130 | 72 | 209 | 171 | 144 | 210 | 231 | 167 | 211 | 86 | 201 |
| 212 | 179 | 21 | 213 | 147 | 42 | 214 | 64 | 84 | 215 | 216 | 168 |
| 216 | 52 | 215 | 217 | 176 | 41 | 218 | 239 | 82 | 219 | 38 | 164 |
| 220 | 55 | 207 | 221 | 12 | 25 | 222 | 17 | 50 | 223 | 68 | 100 |
| 224 | 111 | 200 | 225 | 120 | 23 | 226 | 25 | 46 | 227 | 154 | 92 |
| 228 | 71 | 184 | 229 | 116 | 247 | 230 | 167 | 105 | 231 | 193 | 210 |
| 232 | 35 | 35 | 233 | 83 | 70 | 234 | 137 | 140 | 235 | 251 | 159 |
| 236 | 20 | 185 | 237 | 93 | 245 | 238 | 248 | 109 | 239 | 151 | 218 |
| 240 | 46 | 51 | 241 | 75 | 102 | 242 | 185 | 204 | 243 | 96 | 31 |
| 244 | 15 | 62 | 245 | 237 | 124 | 246 | 62 | 248 | 247 | 229 | 119 |
| 248 | 246 | 238 | 249 | 135 | 91 | 250 | 165 | 182 | 251 | 23 | 235 |
| 252 | 58 | 81 | 253 | 163 | 162 | 254 | 60 | 195 | 255 | 183 | 99 |

Table 2. Coefficients of Generator Polynomial

| | | | | |
|----------|-----|---------------|-----|----------------|
| g_0 | $=$ | g_{32} | $=$ | α^0 |
| g_1 | $=$ | g_{31} | $=$ | α^{249} |
| g_2 | $=$ | g_{30} | $=$ | α^{59} |
| g_3 | $=$ | g_{29} | $=$ | α^{66} |
| g_4 | $=$ | g_{28} | $=$ | α^4 |
| g_5 | $=$ | g_{27} | $=$ | α^{43} |
| g_6 | $=$ | g_{26} | $=$ | α^{126} |
| g_7 | $=$ | g_{25} | $=$ | α^{251} |
| g_8 | $=$ | g_{24} | $=$ | α^{97} |
| g_9 | $=$ | g_{23} | $=$ | α^{30} |
| g_{10} | $=$ | g_{22} | $=$ | α^3 |
| g_{11} | $=$ | g_{21} | $=$ | α^{213} |
| g_{12} | $=$ | g_{20} | $=$ | α^{50} |
| g_{13} | $=$ | g_{19} | $=$ | α^{66} |
| g_{14} | $=$ | g_{18} | $=$ | α^{170} |
| g_{15} | $=$ | g_{17} | $=$ | α^5 |
| g_{16} | $=$ | α^{24} | | |

The algorithm used is a transform decoder as described in [4], which is based on a modified Euclid algorithm to compute the error locator polynomial. Therefore, this simulation can be used to verify the performance of the proposed pipeline hardware decoder.

The only modifications consist in adapting the algorithm to symmetric generator polynomials, using a finite field FFT (Fast Fourier Transform) to compute the error pattern, and testing for uncorrectable error patterns.

2. SIMULATION SET-UP

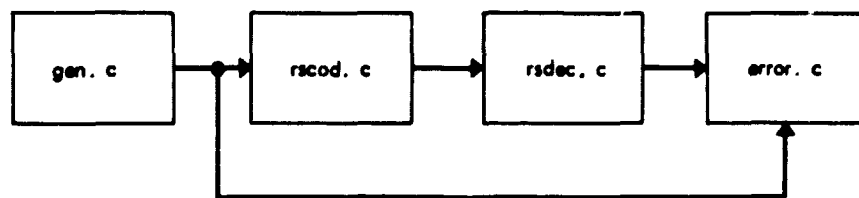
The set of software subroutines includes a random generator (gen.c) of sequences taken from $\{0,1\}$, a RS encoder (rsenc.c), a RS decoder (rsdec.c), and a block (error.c) which computes bit and symbol error probabilities. These subroutines are called by a main program named "universe.c".

All programs are written in C-language on a VAX 750 computer. Figure 1(a) shows the block diagram of the simulation set-up. Channel errors are artificially introduced at the input of the RS decoder. If desired, the set-up may be modified to that of Fig. 1(b), where errors are produced by adding a sequence of random variables (for example Gaussian, if the subroutine "gauss.c" is used) to the encoded stream. Error bursts may be added with a separate subroutine, or by a concatenated, convolutional code and Viterbi decoding.

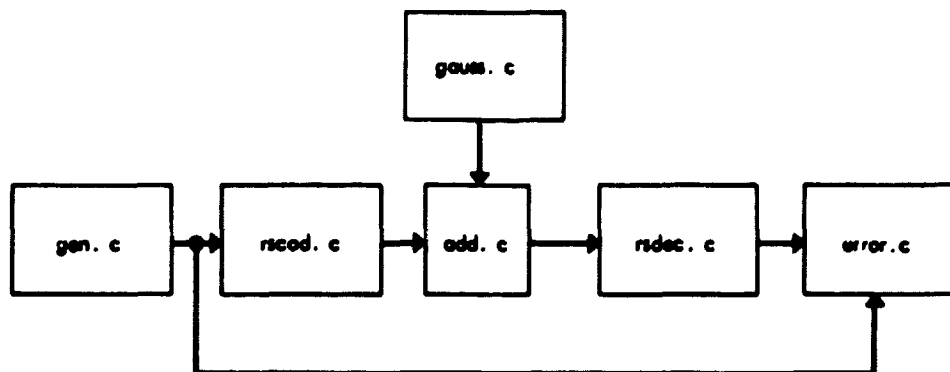
The modularity of the program allows the simulation of concatenated coding schemes to be described in a separate report.

3. RS ENCODER

Since we are considering a systematic RS code, the encoder will first output the K information symbols a_j . The T parity symbols are the coefficients b_i of the remainder polynomial $b(x) = b_0 + b_1 x + \dots + b_{T-1} x^{T-1}$, resulting from dividing the message polynomial $x^T a(x)$ by the generator polynomial $g(x)$, where $a(x) = a_0 + a_1 x + \dots + a_{K-1} x^{K-1}$.



(a) Fixed Error Pattern



(b) Random Error Pattern

Fig. 1. Simulation Block Diagram

This polynomial division can be easily implemented by a shift register divider, as shown in the logic diagram of Fig. 2, for the (255,223) RS code. Additions are to be interpreted modulo-two (exclusive-OR), multiplications in the field are performed by table look-up, where the table is automatically constructed during the first execution. The subroutine listing is shown in Appendix B.

The algorithm proceeds as follows:

- (0) Initialize $b_i = 0, i=0, \dots, T$
- (A) During the first 223 iterations ($0 \leq j < 222$):
 - (1) get information symbol a_j
 - (2) $v = a_j + b_{T-1}$
 - (3) output $z = a_j$
- (B) During last 32 iterations ($224 \leq j < 255$):
 - (1) $v = 0$
 - (2) output $z = b_{T-1}$
- (C) For all j 's:
 - (1) $b_i = b_{i-1} + (g_i * v), i=T-1, T-2, \dots, 1$
 - (2) $b_0 = v$

The encoder may be tested by forcing the generator to produce some given pattern whose corresponding codeword is known, and printing the output

4. RS DECODER

4.1 DECODER ALGORITHM

The decoder performs the following basic operations:

- get received codeword
- compute syndrome
- obtain the error-locator polynomial by using the modified Euclid algorithm
- compute the remaining elements of the error sequence transform
- compute the inverse transform yielding the estimated error pattern.

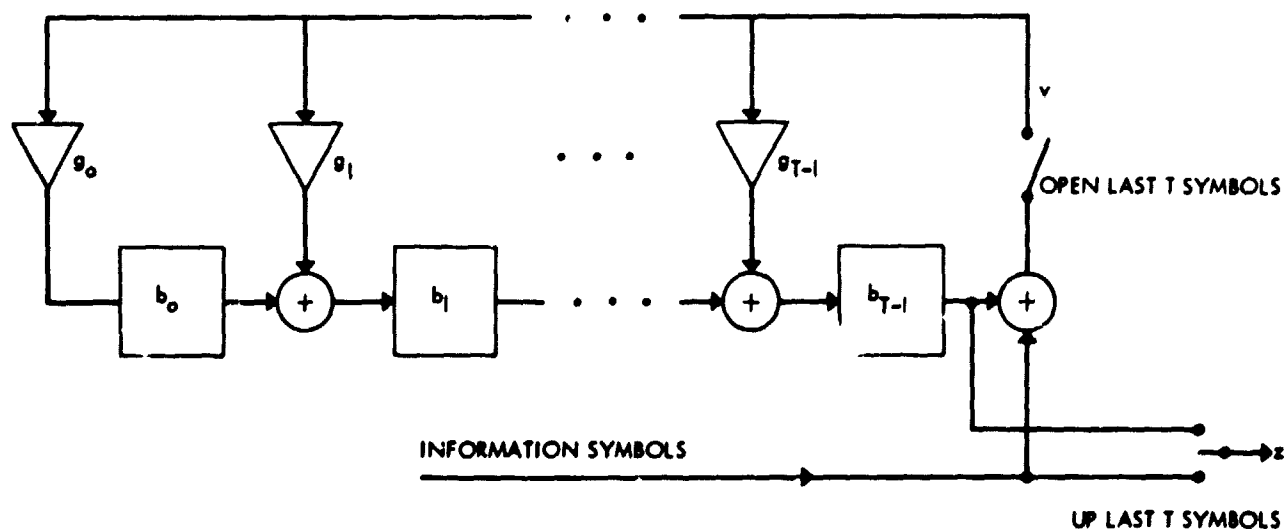


Fig. 2. RS Encoder

Consider the generator polynomial in (1), and define:

$$\begin{aligned} \underline{U} &= [u_0, u_1, \dots, u_{N-1}] && \text{received codeword} \\ \text{where } \begin{cases} [u_0, u_1, \dots, u_{N-T-1}] &= \text{information symbols} \\ [u_{N-T}, \dots, u_{N-1}] &= \text{parity} \end{cases} \\ \underline{S} &= [s_0, s_1, \dots, s_{T-1}] && \text{syndrome} \\ \underline{R} &= [r_0, r_1, \dots, r_T] \\ \underline{\lambda} &= [\lambda_0, \lambda_1, \dots, \lambda_{T-1}] && (\text{contains error-locator polynomial at last iteration}) \\ \underline{\mu} &= [\mu_0, \mu_1, \dots, \mu_{T-1}] \\ \underline{E} &= [E_0, E_1, \dots, E_N] && \text{error pattern transform} \\ \underline{e} &= [e_0, e_1, \dots, e_N] && \text{error pattern} \\ d(\underline{S}) &= \{ j : s_j \neq 0 \text{ and } s_i = 0, j < i < T \} \end{aligned}$$

and similarly for $d(\underline{R})$ and $d(\underline{\lambda})$.

Then the decoder algorithm can be described as follows:

- (1) get received codeword \underline{U}
- (2) compute the syndrome, (see Appendix A)

$$\underline{S} = \underline{0}$$

$$s_j = u_{N-1-j} + \alpha^{G(j+M)} s_j; j=0, \dots, T-1; i=0, \dots, N-1$$
- (3) if $d(\underline{S})=0$ go to (11)
- (4) initialize,

$$E_{j+1} = s_{T-1-j}; j=0, \dots, T-1$$

$$\underline{R} = \underline{0}$$

$$r_T = 1$$

$$\underline{\mu} = \underline{0}$$

$$\mu_0 = 1$$

$$\underline{\lambda} = \underline{0}$$

$$i = 1$$
- (5) while $(i < T)$ do:

$$L = d(\underline{R}) - d(\underline{S})$$
 if $d(\underline{R}) < T/2$ go to (6)
 else if $d(\underline{S}) < T/2, \underline{\lambda} = \underline{\mu}$, go to (6)
 else do EUCLID (see section 4.2)

$$i = i+1$$

- (6) compute normalized error-locator polynomial

$$B = \alpha^{-\lambda_{d(\underline{\lambda})}}$$

$$\lambda_j = B \lambda_j ; j=0, \dots, d(\underline{\lambda})$$

- (7) compute remaining elements of error transform

$$E_{j+1} = 0$$

$$\left. \begin{aligned} E_{j+1} &= E_{j+1} + \lambda_{d(\underline{\lambda})-1-i} E_{j-1} ; i=0, \dots, d(\underline{\lambda})-1 \\ E_0 &= E_n \end{aligned} \right\} j=T, \dots, N+T-1$$

- (8) test for uncorrectable error patterns,

if $E_j \neq E_{j+N}$ for some $j=1, \dots, T$, go to (11)

- (9) compute inverse transform \underline{e} (see section 4.3)

- (10) compute corrected sequence,

$$\underline{U} = \underline{U} + \underline{e}$$

- (11) output \underline{U}

A complete listing of the decoder subroutine is shown in Appendix C.

The test in step (8) is explained in [6]. It may also be observed that this RS code is effective in terms of undetected errors, since [7, 8], for independent symbol errors, the probability of undetected error P_u is bounded by:

$$P_u < (N+1)^{-T} \sum_{i=0}^{T/2} \binom{N}{i} N^i < \frac{1}{(T/2)!}$$

For the (255,223) code, $P_u < 4.8 \cdot 10^{-14}$. But, for the (15,9) code considered in [4], $P_u < 0.093$.

4.2 EUCLID ALGORITHM

This is a modified version of Euclid's algorithm for polynomials [5], which does not need the computation of inverse field elements. It operates on two polynomials,

$$A(x) = x^T \quad \text{and} \quad S(x) = \sum_{K=1}^T a_K x^{T-K}$$

and finds the i^{th} remainder $R_i(x)$ of degree less than $T/2$, satisfying:

$$\gamma_i(x) A(x) + \lambda_i(x) S(x) = R_i(x)$$

At the end, $\lambda_j(x)$ is the desired (unnormalized) locator polynomial. The algorithm is implemented as follows:

$$\begin{aligned} & \text{if } d(\underline{R}) < d(\underline{S}) \left\{ \begin{array}{l} \underline{R} \longleftrightarrow \underline{S} \\ \underline{\lambda} \longleftrightarrow \underline{\mu} \\ d(\underline{R}) \longleftrightarrow d(\underline{S}) \end{array} \right. \\ & \text{if } s_{d(\underline{S})} = 0 \left\{ \begin{array}{l} d(\underline{S}) \longleftarrow d(\underline{S}) - 1 \\ \text{if } d(\underline{S}) < T/2, \underline{\lambda} = \underline{\mu}, \text{ return} \end{array} \right. \\ & \text{else} \left\{ \begin{array}{l} a = r_{d(\underline{R})} ; b = s_{d(\underline{S})} \\ d(\underline{R}) \longleftarrow d(\underline{R}) - 1 \\ \hat{\underline{S}} = D_{|L|}(\underline{S}) \\ \underline{R} = b \underline{R} + a \hat{\underline{S}} \\ \hat{\underline{\mu}} = D_{|L|}(\underline{\mu}) \\ \underline{\lambda} = b \underline{\lambda} + a \hat{\underline{\mu}} \\ \text{if } d(\underline{R}) < T/2, \text{ return} \end{array} \right. \end{aligned}$$

where $D_{|L|}(\underline{x})$ shifts right the components of a vector \underline{x} by $|L|$ positions, and fills with zeros.

4.3 INVERSE FFT

A direct computation of the inverse transform,

$$e_j = \sum_{i=0}^{N-1} \alpha^{Gij} E_{N+1+i+jM}; \quad i=0, \dots, N-1; \quad j=0, \dots, N-1$$

requires $N^2 = 65025$ multiplications. The number of multiplications may be reduced by organizing the N -point one-dimensional array \underline{E} into a two-dimensional $n_1 \times n_2$ array, where $n_1 n_2 = N$, and n_1 and n_2 are

relatively prime. This algorithm (Good-Thomas FFT [6]) is based on the Chinese remainder theorem.

Let $b = (a)_N$ denote the remainder of a modulo N , and define

$$i_1 = (i)_{n_1}, i_2 = (i)_{n_2}, j_1 = (\tilde{N}j)_{n_1}, \text{ and } j = (\tilde{N}j)_{n_2}$$

Then,

$$i = (\tilde{N}(n_2 i_1 + n_1 i_2))_N \quad \text{and} \quad j = (n_2 j_1 + n_1 j_2)_N$$

where, $(\tilde{N}(n_1 + n_2)) = 1 \implies \tilde{N} = 8.$

Now the inverse transform may be written in the following two steps

$$D_{i_1, j_2} = \sum_{i_2=0}^{n_2-1} E_{N+1-M+1}^{\alpha} G_{n_1 i_2 j_2} \quad 0 < i_1 < n_1, 0 < j_2 < n_2$$

$$e_j = \sum_{i_1=0}^{n_1-1} D_{i_1, j_2}^{\alpha} G_{n_2 i_1 j_1} \quad 0 < j_1 < n_1, 0 < j_2 < n_2$$

For $N = 255 = 17 \cdot 15 = n_1 n_2$, the number of multiplications is reduced from N^2 to $N(n_1 + n_2)$. A further reduction may be obtained, if desired, by factoring N as $N = 17 \cdot 5 \cdot 3$.

5. USER GUIDE AND EXAMPLES

This software package may be run on any computer having a C-language compiler. The source code for the full set of subroutines is available by contacting the author. Subroutines required are:

(block management routines in object code form)

sequencer.o

block.o

fifo.o

(include files)

star.h
dstar.h
type.h
alloc.h
para.h
param.h
dfifo.h

(simulation blocks)

gen.c
rscod.c
rsdec.c
add.c
gauss.c
error.c
universe.c

Subroutines are also available to simulate the (15,9) RS code considered in [4]. If the UNIX operating system is used, it is advisable to create a "makefile" to maintain (compile and link) the set of subroutines. In any case the subroutines must be compiled and linked to produce an executable image file.

In order to run the simulation it is not necessary to provide any external parameter or data file, since the information symbols are generated internally. If specific information sequences are of interest, the subroutine "gen.c" can be easily modified for this purpose. Non-real-time decoding of actual data could be accomplished by modifying "rsdec.c" so that it will read the data from a disk file in segments of a given number of blocks.

The output contains the number of channel symbol errors, and the number of corrected symbol and bit errors. If the number of channel errors is greater than $T/2$, a warning message is printed. If real data needs to be

decoded, the decoded symbols can be displayed by adding a print statement in "rsdec.c". All output is normally displayed on the standard output (CRT), but it can be redirected to a disk file through the operating system. As an example, under UNIX, we could type:

```
sim > outfile
```

where "sim" is the executable program and the file "outfile" will contain the output.

By including the print statements provided in the subroutine rsdec.c, it is possible to display all the intermediate steps of the decoder. Such an example of output is shown in Appendix D for a given codeword and the error pattern $e_7 = \alpha^{202}$, $e_{120} = \alpha^0$, $e_i = 0$, $i \neq 7, 120$. Elements in GF(256) are represented in decimal base.

If no errors were present, the output would show that S = 0.

A randomly chosen codeword is shown in Appendix E.

6.

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APPENDIX A

Number of Multiplications Required to Compute the Syndrome of a (255,223)
RS Code with a Symmetric Generator Polynomial

Consider the generator polynomial in (1), then the syndrome is defined as,

$$s_j = \sum_{i=0}^{N-1} u_i \alpha^{Gi(j+M)}, \quad j=0, \dots, T$$

Define the $N \times N$ matrix J ,

$$J = \begin{bmatrix} 0 & & & 1 \\ & \ddots & & \\ & & \ddots & \\ 1 & & & 0 \end{bmatrix}$$

such that,

$$\tilde{u} = [\tilde{u}_0, \dots, \tilde{u}_{N-1}] = [u_{N-1}, \dots, u_0] = \underline{u} J$$

Then we can consider a new syndrome \tilde{s}

$$\begin{aligned} \tilde{s}_m &= \sum_{i=0}^{N-1} \tilde{u}_i \alpha^{Gi(m+M)}, \quad m=0, \dots, T-1 \\ &= \sum_{i=0}^{N-1} u_{N-1-i} \alpha^{Gi(m+M)}, \quad m=0, \dots, T-1 \end{aligned}$$

Let $k = N - i - 1$

$$\tilde{s}_m = \sum_{k=0}^{N-1} u_k \alpha^{G(N-1-k)(m+M)} = \alpha^{-G(m+M)} \sum_{k=0}^{N-1} u_k \alpha^{-G(m+M)}$$

But,

$$\alpha^{-G(m+M)} = \alpha^{G(h+M)}, \quad \text{where } m = T - 1 - h, \quad h=0, \dots, T-1$$

$$\rightarrow \tilde{s}_{T-1-h} = \alpha^{G(h+M)} \sum_{k=0}^{N-1} u_k \alpha^{GK(h+M)}, \quad h=0, \dots, T-1$$

And finally, $\tilde{s}_{T-1-h} = \alpha^{G(h+M)} s_h, h=0, \dots, T-1$

$$\tilde{S}J = \underline{S}\Gamma, \text{ where } \Gamma = \begin{bmatrix} \alpha^{GM} & \alpha^{G(M+1)} & 0 & & \\ & \cdot & \cdot & \cdot & \\ 0 & & & & \alpha^{G(M+T-1)} \end{bmatrix}$$

$$\underline{S} = \underline{u}A,$$

where $A = [a_{ij}]$, $a_{ij} = \alpha^{Gi(j+M)}$, $i=0, \dots, N-1$, $j=0, \dots, T-1$

$$\tilde{S} = \underline{u}JA$$

$$\underline{S}(J+\Gamma) = \underline{u}AJ + \underline{u}JAJ = \underline{u}(AJ + JAJ) \triangleq \underline{d}$$

Let

$$B = AJ + JAJ = \begin{bmatrix} \underline{b}_0 & \dots & \underline{b}_j & \dots & \underline{b}_{T-1} \end{bmatrix} \triangleq \left[\begin{array}{c|c} \dots & \begin{bmatrix} \underline{b}_j^* \\ \underline{b}_j^* \\ J\underline{b}_j^* \end{bmatrix} \\ \hline \dots & \dots \end{array} \right]$$

be a partition of B into the column vectors \underline{b}_j , and

$$\underline{u} = [\underline{u}_1, \underline{u}_c, \underline{u}_u]$$

Then
$$\underline{d}_j = \underline{u}_1 \underline{b}_j^* + \underline{u}_c \underline{b}_j^* + \underline{u}_u J \underline{b}_j^* = (\underline{u}_1 + \underline{u}_u J) \underline{b}_j^* + \underline{u}_c \underline{b}_j^*$$

The computation of \underline{d}_j requires only $254/2 + 1 = 128$ multiplications (instead of 255).

$$\underline{S} = \underline{d} (J+\Gamma)^{-1}$$

Note that $(J+\Gamma)^{-1}$ has the form:

$$(J+\Gamma)^{-1} = \begin{bmatrix} \beta_0 & & & 0 & & \epsilon_{T-1} \\ & \beta_1 & & & & \\ & 0 & \cdot & \cdot & & 0 \\ & & \cdot & \cdot & \cdot & \\ \epsilon_1 & & & & & \\ \epsilon_0 & & & 0 & & \beta_{T-1} \end{bmatrix}$$

Therefore,

$$S_j = \beta_j d_j + \epsilon_j d_{T-1-j}, j=0, \dots, T-1$$

So that each S_j can be computed with $128 + 2 = 130$ multiplications.

APPENDIX B

RS Encoder Subroutine

Reed-Solomon Encoder (CCSDS Doc. #1 , Sept 1983) ***/
 /*****

```
#include <stdio.h>
#include "../type.h"
#include "../star.h"
#define TT      32
#define N       255
#define K       223
#define CNT     pstate->cnt
#define V       pstate->vv
#define G       pstate->g
#define B       pstate->b
#define H       pstate->h
#define F       pstate->f
typedef struct {
    int non;
}
PARAM, *PARAMPTR;
typedef struct {
    unsigned char b[TT],g[TT+1],cnt,h[N],f[N+1],vv;
}
STATE, *STATEPTR;
rscode (pparam,size,pstate,psstar)
PARAMPTR pparam;
STATEPTR pstate;
STARPTR psstar;
int size;
{
    SAMPLE x;
    int i,j;

    if (pstate == NULL) {
        pstate = (STATEPTR) alloc_state_var(1,sizeof(STATE));
        if (no_input_fifos( ) !=1 || no_output_fifos( ) !=2)
            return(3);
    }

    /*** H[ ] and F[ ] compute the power and log in GF(256) *****/

    H[0]=1;
    for(i=0;i<8;i++) H[i+1]=2*H[i];
    for(i=8;i<N;i++) H[i]=H[i-1]^H[i-6]^H[i-7]^H[i-8];

    for(j=1;j<N+1;j++) {
        for(i=0;i<N;i++) {
            if(H[i]==j) F[j]=i;
        }
    }

    CNT=0;
    V=0;
    F[0]=0;
```

```

/**** G[ ] are the coefficients of the generating polynomial ****/

```

```

G[0]=H[0];
G[1]=H[249];
G[2]=H[59];
G[3]=H[66];
G[4]=H[4];
G[5]=H[43];
G[6]=H[126];
G[7]=H[251];
G[8]=H[97];
G[9]=H[30];
G[10]=H[3];
G[11]=H[213];
G[12]=H[50];
G[13]=H[66];
G[14]=H[170];
G[15]=H[5];
G[16]=H[24];
for(i=0;i<TT/2;i++) G[TT-1]=G[i];

```

```

if(length_output_fifo(0) != length_output_fifo(1)) return(7);
if (length_output_fifo(0)==maxlength_output_fifo(0)) return(0);
if (length_output_fifo(1)==maxlength_output_fifo(1)) return(0);

```

```

while(length_input_fifo(0) >0 || CNT>=K )
{

```

```

if(length_output_fifo(0)==maxlength_output_fifo(0)) return(0);
if(length_output_fifo(1)==maxlength_output_fifo(1)) return(0);

```

```

if(CNT==0) for(i=0;i<TT;i++) B[i]=0;

```

```

/*****

```

```

if(CNT<K) /** information bits **/

```

```

{
    get(0,&x) ;
    V=((int)x)^B[TT-1];
}

```

```

else /** parity bits **/

```

```

{
    V=0;
    x=(SAMPLE)B[TT-1];
}

```

```

for(i=TT-1;i>0;i--)
B[i]=B[i-1]^(V!=0)*(H[(F[G[i]]+F[V])%N]);

```

```

B[0]=(V!=0)*(H[(F[G[0]]+F[V])%N]);

```

```

/*****

```

```

put(0,x);
put(1,x);

```

```

CNT=(CNT+1)%N;

```

```

}
return (0) ;

```

```

}

```

APPENDIX C

RS Decoder Subroutine

/****** Reed-Solomon Decoder *****/

```
#include <stdio.h>
#include "../type.h"
#include "../star.h"
#define TT      32
#define N       255
#define M       112
#define G       11
#define H       pstate->h
#define F       pstate->f
#define MUL(A, B) ((B!=0)*(H[(A+F[(B))%N]))
typedef struct {
    int non;
}
PARAM, *PARAMPTR;
typedef struct {
    unsigned char h[N],f[N+1];
}
STATE, *STATEPTR;
rsdec (pparam,size,pstate,pstar)
PARAMPTR pparam;
STATEPTR pstate;
STARPTR pstar;
int size;
{
    SAMPLE x;
    unsigned char et[17],ex[N],e[N],E[N+TT+1],S[TT+1],degR,degS;
    unsigned char R[TT+1],mu[TT+1],lam[TT+1],REC[N],tem,fa,fb;
    unsigned char *PR,*PS,*PT,*Plam,*Pmu,a,b;
    int i,j,L,CL,TH,ix,jx,i1,i2,j1,j2;

    if (pstate == NULL) {
        pstate = (STATEPTR) alloc_state_var(1,sizeof(STATE));
        if (no_input_fifos( ) !=1 || no_output_fifos( ) !=1)
            return(3);
    }

    /* H[ ] and F[ ] compute the power and log in GF(256) *****/

    H[0]=1;
    for(i=0;i<8;i++) H[i+1]=2*H[i];
    for(i=8;i<N;i++) H[i]=H[i-1]^H[i-6]^H[i-7]^H[i-8];

    for(j=1;j<N+1;j++) {
        for(i=0;i<N;i++) {
            if(H[i]==j) F[j]=i;
        }
    }

    F[0]=0;
}
```

```

if (length_output_fifo(0)==maxlength_output_fifo(0)) return(0);

    while(length_input_fifo(0) >0 )
    {
if (length_output_fifo(0)==maxlength_output_fifo(0)) return(0);
/*****
for(j=0;j<N;j++)
{
get(0,&x);
REC[j]=(char)x;
e[j]=0;
}

PR=R;
PS=S;
Plam=lam;
Pmu=mu;
for(j=0;j<=TT;j++) { R[j]=0; S[j]=0; lam[j]=0; mu[j]=0; }
/***** Syndrome calculation *****/
for(i=0;i<N;i++)
{
ix=N-1-i;
for(j=0;j<TT;j++) S[j]=REC[ix]^MUL(G*(j+M),S[j]);
}
degS=TT;
while(*(degS+PS)==0 && degS>0) --degS;
if(degS>0)
{

/***** Modified Euclid Algorithm *****/
for(j=0;j<TT;j++) E[j+1]= *(PS+TT-1-j);
*(PR+TT)=1;
*mu=1;
degR=TT;
degS=TT;
i=1;
TH=TT/2;

while(i<=TT)
{
while(*(degR+PR)==0 && degR>0) --degR;
while(*(degS+PS)==0 && degS>0) --degS;

L=degR-degS;
CL=L;
if(L<0) CL= -L;

if(degR<TH || degS<TH)
{
if(degR>=TH) Plam=Pmu;
break;
}

else

```

```

{
  if(degR<degS)
  {
    PT=PR;
    PR=PS;
    PS=PT;
    PT=Plam;
    Plam=Pmu;
    Pmu=PT;
    tem=degR;
    degR=degS;
    degS=tem;
  }
  if(*(PS+degS)==0)
  {
    degS--;
    if(degS<TH)
    {
      Plam=Pmu;
      break;
    }
  }
  else
  {
    /* compute R lam *****/
    a= *(PR+degR);
    b= *(PS+degS);
    fa=F[a];
    fb=F[b];
    degR--;
    for(j=0;j<=TT;j++)
    {
      tem=(j>=CL)*(*(PS+j-CL));
      PT=PR+j;
      *PT=(b!=0)*MUL(fb,*PT)^(a!=0)*MUL(fa,tem);
      tem=(j>=CL)*(*(Pmu+j-CL));
      PT=Plam+j;
      *PT=(b!=0)*MUL(fb,*PT)^(a!=0)*MUL(fa,tem);
    }
    /* *****/

    if(degR<TH) break;
  }
  i++;
}
/***** Error locator polynomial *****/
degR=TT;
while(*(degR+Plam)==0 && degR>0) --degR;

tem=N-F[*(Plam+degR)];
for(j=0;j<=degR;j++)
{
  PT=Plam+j;
  *PT=MUL(tem,*PT);
}

```

```

}
/*****
for(j=TT; j<N+TT; j++)
{
    E[j+1]=0;
    for(i=0; i<degR; i++)
    {
        tem= *(Plam+degR-i-1);
        jx=j-1;
        if(tem!=0) E[j+1] ^= MUL(F[tem], E[jx]);
    }
}
E[0]=E[N];

for(j=1; j<=TT; j++)
if(E[j] != E[j+N]) {printf("ln * TEST FAILED ***"); j=0; break;}
if(j!=0)
{

/***** Inverse FFT *****/
for(j2=0; j2<15; j2++)
{
    jx=G*17*j2;
    for(i1=0; i1<17; i1++)
    {
        et[i1]=0;
        for(i2=0; i2<15; i2++)
        {
            i=(N+1-M+8*(15*i1+17*i2))%N;
            et[i1] ^= MUL(jx*i2, E[i]);
        }
    }

    for(j1=0; j1<17; j1++)
    {
        ix=G*15*j1;
        j=(15*j1+17*j2)%N;
        e[j]=0;
        for(i1=0; i1<17; i1++)
        {
            e[j] ^= MUL(ix*i1, et[i1]);
        }
    }

    for(j=0; j<N; j++) REC[j] ^= e[j];
}
}
for(j=0; j<N; j++)
{
    x=(SAMPLE)REC[j];
    put(0,x);
}
return (0) ;

```

APPENDIX D

Example of Output

| i | u | i | u | i | u | i | u |
|-----|---|-----|---|-----|---|-----|---|
| 0 | 0 | 1 | 0 | 2 | 0 | 3 | 0 |
| 4 | 0 | 5 | 0 | 6 | 0 | 7 | 0 |
| 8 | 0 | 9 | 0 | 10 | 0 | 11 | 0 |
| 12 | 0 | 13 | 0 | 14 | 0 | 15 | 0 |
| 16 | 0 | 17 | 0 | 18 | 0 | 19 | 0 |
| 20 | 0 | 21 | 0 | 22 | 0 | 23 | 0 |
| 24 | 0 | 25 | 0 | 26 | 0 | 27 | 0 |
| 28 | 0 | 29 | 0 | 30 | 0 | 31 | 0 |
| 32 | 0 | 33 | 0 | 34 | 0 | 35 | 0 |
| 36 | 0 | 37 | 0 | 38 | 0 | 39 | 0 |
| 40 | 0 | 41 | 0 | 42 | 0 | 43 | 0 |
| 44 | 0 | 45 | 0 | 46 | 0 | 47 | 0 |
| 48 | 0 | 49 | 0 | 50 | 0 | 51 | 0 |
| 52 | 0 | 53 | 0 | 54 | 0 | 55 | 0 |
| 56 | 0 | 57 | 0 | 58 | 0 | 59 | 0 |
| 60 | 0 | 61 | 0 | 62 | 0 | 63 | 0 |
| 64 | 0 | 65 | 0 | 66 | 0 | 67 | 0 |
| 68 | 0 | 69 | 0 | 70 | 0 | 71 | 0 |
| 72 | 0 | 73 | 0 | 74 | 0 | 75 | 0 |
| 76 | 0 | 77 | 0 | 78 | 0 | 79 | 0 |
| 80 | 0 | 81 | 0 | 82 | 0 | 83 | 0 |
| 84 | 0 | 85 | 0 | 86 | 0 | 87 | 0 |
| 88 | 0 | 89 | 0 | 90 | 0 | 91 | 0 |
| 92 | 0 | 93 | 0 | 94 | 0 | 95 | 0 |
| 96 | 0 | 97 | 0 | 98 | 0 | 99 | 0 |
| 100 | 0 | 101 | 0 | 102 | 0 | 103 | 0 |
| 104 | 0 | 105 | 0 | 106 | 0 | 107 | 0 |
| 108 | 0 | 109 | 0 | 110 | 0 | 111 | 0 |
| 112 | 0 | 113 | 0 | 114 | 0 | 115 | 0 |
| 116 | 0 | 117 | 0 | 118 | 0 | 119 | 0 |
| 120 | 0 | 121 | 0 | 122 | 0 | 123 | 0 |
| 124 | 0 | 125 | 0 | 126 | 0 | 127 | 0 |
| 128 | 0 | 129 | 0 | 130 | 0 | 131 | 0 |
| 132 | 0 | 133 | 0 | 134 | 0 | 135 | 0 |
| 136 | 0 | 137 | 0 | 138 | 0 | 139 | 0 |
| 140 | 0 | 141 | 0 | 142 | 0 | 143 | 0 |
| 144 | 0 | 145 | 0 | 146 | 0 | 147 | 0 |
| 148 | 0 | 149 | 0 | 150 | 0 | 151 | 0 |
| 152 | 0 | 153 | 0 | 154 | 0 | 155 | 0 |
| 156 | 0 | 157 | 0 | 158 | 0 | 159 | 0 |
| 160 | 0 | 161 | 0 | 162 | 0 | 163 | 0 |
| 164 | 0 | 165 | 0 | 166 | 0 | 167 | 0 |
| 168 | 0 | 169 | 0 | 170 | 0 | 171 | 0 |
| 172 | 0 | 173 | 0 | 174 | 0 | 175 | 0 |
| 176 | 0 | 177 | 0 | 178 | 0 | 179 | 0 |
| 180 | 0 | 181 | 0 | 182 | 0 | 183 | 0 |
| 184 | 0 | 185 | 0 | 186 | 0 | 187 | 0 |
| 188 | 0 | 189 | 0 | 190 | 0 | 191 | 0 |
| 192 | 0 | 193 | 0 | 194 | 0 | 195 | 0 |
| 196 | 0 | 197 | 0 | 198 | 0 | 199 | 0 |

(Codeword)

| | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 200 | 0 | 201 | 0 | 202 | 0 | 203 | 0 |
| 204 | 0 | 205 | 0 | 206 | 0 | 207 | 0 |
| 208 | 0 | 209 | 0 | 210 | 0 | 211 | 0 |
| 212 | 0 | 213 | 0 | 214 | 0 | 215 | 0 |
| 216 | 0 | 217 | 0 | 218 | 0 | 219 | 0 |
| 220 | 0 | 221 | 0 | 222 | 255 | 223 | 53 |
| 224 | 204 | 225 | 91 | 226 | 198 | 227 | 46 |
| 228 | 110 | 229 | 212 | 230 | 226 | 231 | 42 |
| 232 | 99 | 233 | 17 | 234 | 70 | 235 | 91 |
| 236 | 194 | 237 | 11 | 238 | 36 | 239 | 11 |
| 240 | 194 | 241 | 91 | 242 | 70 | 243 | 17 |
| 244 | 99 | 245 | 42 | 246 | 226 | 247 | 212 |
| 248 | 110 | 249 | 46 | 250 | 198 | 251 | 91 |
| 252 | 204 | 253 | 53 | 254 | 255 | | |

No. of input errors = 2

1 s log()

| | | | |
|----|-----|-----|------------|
| 0 | 16 | 4 | (Syndrome) |
| 1 | 117 | 136 | |
| 2 | 121 | 184 | |
| 3 | 191 | 134 | |
| 4 | 113 | 24 | |
| 5 | 55 | 53 | |
| 6 | 187 | 74 | |
| 7 | 243 | 96 | |
| 8 | 248 | 246 | |
| 9 | 218 | 239 | |
| 10 | 100 | 223 | |
| 11 | 66 | 88 | |
| 12 | 233 | 83 | |
| 13 | 30 | 43 | |
| 14 | 170 | 85 | |
| 15 | 151 | 145 | |
| 16 | 58 | 33 | |
| 17 | 190 | 22 | |
| 18 | 238 | 248 | |
| 19 | 33 | 87 | |
| 20 | 130 | 156 | |
| 21 | 254 | 60 | |
| 22 | 209 | 171 | |
| 23 | 133 | 159 | |
| 24 | 165 | 30 | |
| 25 | 188 | 182 | |
| 26 | 21 | 212 | |
| 27 | 107 | 63 | |
| 28 | 241 | 75 | |
| 29 | 74 | 173 | |
| 30 | 37 | 172 | |
| 31 | 16 | 4 | |

i= 1
d(R)= 32 d(S)= 31 L= 1
a= 0 b= 4

| i | s | log() | r | log() | log() | u | log() |
|----|-----|-------|-----|-------|-------|---|-------|
| 0 | 16 | 4 | 0 | * | 0 | 1 | 0 |
| 1 | 117 | 136 | 16 | 4 | 1 | 0 | * |
| 2 | 121 | 184 | 117 | 136 | 0 | 0 | * |
| 3 | 191 | 134 | 121 | 184 | 0 | 0 | * |
| 4 | 113 | 24 | 191 | 134 | 0 | 0 | * |
| 5 | 55 | 53 | 113 | 24 | 0 | 0 | * |
| 6 | 187 | 74 | 55 | 53 | 0 | 0 | * |
| 7 | 243 | 96 | 187 | 74 | 0 | 0 | * |
| 8 | 248 | 246 | 243 | 96 | 0 | 0 | * |
| 9 | 218 | 239 | 248 | 246 | 0 | 0 | * |
| 10 | 100 | 223 | 218 | 239 | 0 | 0 | * |
| 11 | 66 | 88 | 100 | 223 | 0 | 0 | * |
| 12 | 233 | 83 | 66 | 88 | 0 | 0 | * |
| 13 | 30 | 43 | 233 | 83 | 0 | 0 | * |
| 14 | 170 | 85 | 30 | 43 | 0 | 0 | * |
| 15 | 151 | 145 | 170 | 85 | 0 | 0 | * |
| 16 | 58 | 33 | 151 | 145 | 0 | 0 | * |
| 17 | 190 | 22 | 58 | 33 | 0 | 0 | * |
| 18 | 238 | 248 | 190 | 22 | 0 | 0 | * |
| 19 | 33 | 87 | 238 | 248 | 0 | 0 | * |
| 20 | 130 | 156 | 33 | 87 | 0 | 0 | * |
| 21 | 254 | 60 | 130 | 156 | 0 | 0 | * |
| 22 | 209 | 171 | 254 | 60 | 0 | 0 | * |
| 23 | 133 | 159 | 209 | 171 | 0 | 0 | * |
| 24 | 165 | 30 | 133 | 159 | 0 | 0 | * |
| 25 | 188 | 182 | 165 | 30 | 0 | 0 | * |
| 26 | 21 | 212 | 188 | 182 | 0 | 0 | * |
| 27 | 107 | 63 | 21 | 212 | 0 | 0 | * |
| 28 | 241 | 75 | 107 | 63 | 0 | 0 | * |
| 29 | 74 | 173 | 241 | 75 | 0 | 0 | * |
| 30 | 37 | 172 | 74 | 173 | 0 | 0 | * |
| 31 | 16 | 4 | 37 | 172 | 0 | 0 | * |
| 32 | 0 | * | 0 | * | 0 | 0 | * |

i= 2

d(R)= 31 d(S)= 31 L= 0

a=172 b= 4

| | | | | | | | | |
|----|-----|-----|-----|-----|----|-----|---|---|
| 0 | 16 | 4 | 217 | 176 | 37 | 172 | 1 | 0 |
| 1 | 117 | 136 | 176 | 192 | 16 | 4 | 0 | * |
| 2 | 121 | 184 | 199 | 204 | 0 | * | 0 | * |
| 3 | 191 | 134 | 103 | 47 | 0 | * | 0 | * |
| 4 | 113 | 24 | 240 | 46 | 0 | * | 0 | * |
| 5 | 55 | 53 | 156 | 133 | 0 | * | 0 | * |
| 6 | 187 | 74 | 134 | 27 | 0 | * | 0 | * |
| 7 | 243 | 96 | 46 | 226 | 0 | * | 0 | * |
| 8 | 248 | 246 | 251 | 23 | 0 | * | 0 | * |
| 9 | 218 | 239 | 52 | 128 | 0 | * | 0 | * |
| 10 | 100 | 223 | 212 | 179 | 0 | * | 0 | * |
| 11 | 66 | 88 | 124 | 245 | 0 | * | 0 | * |
| 12 | 233 | 83 | 180 | 150 | 0 | * | 0 | * |
| 13 | 30 | 43 | 137 | 9 | 0 | * | 0 | * |

| | | | | | | | | |
|----|-----|-----|-----|-----|---|---|---|---|
| 14 | 170 | 85 | 99 | 186 | 0 | * | 0 | * |
| 15 | 151 | 145 | 114 | 70 | 0 | * | 0 | * |
| 16 | 58 | 33 | 83 | 138 | 0 | * | 0 | * |
| 17 | 190 | 22 | 231 | 193 | 0 | * | 0 | * |
| 18 | 238 | 248 | 185 | 236 | 0 | * | 0 | * |
| 19 | 33 | 87 | 65 | 155 | 0 | * | 0 | * |
| 20 | 130 | 156 | 7 | 106 | 0 | * | 0 | * |
| 21 | 254 | 60 | 174 | 37 | 0 | * | 0 | * |
| 22 | 209 | 171 | 148 | 174 | 0 | * | 0 | * |
| 23 | 173 | 159 | 202 | 77 | 0 | * | 0 | * |
| 24 | 115 | 30 | 173 | 11 | 0 | * | 0 | * |
| 25 | 188 | 182 | 119 | 247 | 0 | * | 0 | * |
| 26 | 21 | 212 | 11 | 188 | 0 | * | 0 | * |
| 27 | 107 | 63 | 72 | 208 | 0 | * | 0 | * |
| 28 | 241 | 75 | 219 | 38 | 0 | * | 0 | * |
| 29 | 74 | 173 | 169 | 146 | 0 | * | 0 | * |
| 30 | 37 | 172 | 177 | 115 | 0 | * | 0 | * |
| 31 | 16 | 4 | 0 | * | 0 | * | 0 | * |
| 32 | 0 | * | 0 | * | 0 | * | 0 | * |

i= 3
d(R)= 30 d(S)= 31 L= -1
a= 4 b=115

| | | | | | | | | |
|----|-----|-----|-----|-----|----|-----|-----|-----|
| 0 | 179 | 119 | 217 | 176 | 37 | 172 | 177 | 115 |
| 1 | 196 | 41 | 176 | 192 | 16 | 4 | 217 | 176 |
| 2 | 159 | 235 | 199 | 204 | 0 | * | 135 | 8 |
| 3 | 19 | 78 | 103 | 47 | 0 | * | 0 | * |
| 4 | 202 | 77 | 240 | 46 | 0 | * | 0 | * |
| 5 | 125 | 164 | 156 | 133 | 0 | * | 0 | * |
| 6 | 252 | 58 | 134 | 27 | 0 | * | 0 | * |
| 7 | 4 | 2 | 46 | 226 | 0 | * | 0 | * |
| 8 | 110 | 54 | 251 | 23 | 0 | * | 0 | * |
| 9 | 133 | 159 | 52 | 128 | 0 | * | 0 | * |
| 10 | 167 | 210 | 212 | 179 | 0 | * | 0 | * |
| 11 | 95 | 21 | 124 | 245 | 0 | * | 0 | * |
| 12 | 94 | 181 | 180 | 150 | 0 | * | 0 | * |
| 13 | 98 | 40 | 137 | 9 | 0 | * | 0 | * |
| 14 | 41 | 217 | 99 | 186 | 0 | * | 0 | * |
| 15 | 12 | 101 | 114 | 70 | 0 | * | 0 | * |
| 16 | 150 | 169 | 83 | 138 | 0 | * | 0 | * |
| 17 | 200 | 224 | 231 | 193 | 0 | * | 0 | * |
| 18 | 221 | 12 | 185 | 236 | 0 | * | 0 | * |
| 19 | 99 | 186 | 65 | 155 | 0 | * | 0 | * |
| 20 | 234 | 137 | 7 | 106 | 0 | * | 0 | * |
| 21 | 223 | 68 | 174 | 37 | 0 | * | 0 | * |
| 22 | 9 | 205 | 148 | 174 | 0 | * | 0 | * |
| 23 | 28 | 108 | 202 | 77 | 0 | * | 0 | * |
| 24 | 15 | 42 | 173 | 11 | 0 | * | 0 | * |
| 25 | 251 | 23 | 119 | 247 | 0 | * | 0 | * |
| 26 | 164 | 219 | 11 | 188 | 0 | * | 0 | * |
| 27 | 218 | 235 | 72 | 208 | 0 | * | 0 | * |
| 28 | 57 | 69 | 219 | 38 | 0 | * | 0 | * |
| 29 | 53 | 177 | 169 | 146 | 0 | * | 0 | * |
| 30 | 169 | 146 | 177 | 115 | 0 | * | 0 | * |

| | | | | | | | | |
|----|---|---|---|---|---|---|---|---|
| 31 | 0 | * | 0 | * | 0 | * | 0 | * |
| 32 | 0 | * | 0 | * | 0 | * | 0 | * |

i= 4
d(R)= 30 d(S)= 30 L= 0
a=146 b=115

| | | | | | | | | |
|----|-----|----|-----|-----|----|-----|-----|-----|
| 0 | 32 | 5 | 217 | 176 | 37 | 172 | 2 | 1 |
| 1 | 107 | 63 | 176 | 192 | 16 | 4 | 227 | 154 |
| 2 | 0 | * | 199 | 204 | 0 | * | 147 | 123 |
| 3 | 0 | * | 103 | 47 | 0 | * | 0 | * |
| 4 | 0 | * | 240 | 46 | 0 | * | 0 | * |
| 5 | 0 | * | 156 | 133 | 0 | * | 0 | * |
| 6 | 0 | * | 134 | 27 | 0 | * | 0 | * |
| 7 | 0 | * | 46 | 226 | 0 | * | 0 | * |
| 8 | 0 | * | 251 | 23 | 0 | * | 0 | * |
| 9 | 0 | * | 52 | 128 | 0 | * | 0 | * |
| 10 | 0 | * | 212 | 179 | 0 | * | 0 | * |
| 11 | 0 | * | 124 | 245 | 0 | * | 0 | * |
| 12 | 0 | * | 180 | 150 | 0 | * | 0 | * |
| 13 | 0 | * | 137 | 9 | 0 | * | 0 | * |
| 14 | 0 | * | 99 | 186 | 0 | * | 0 | * |
| 15 | 0 | * | 114 | 70 | 0 | * | 0 | * |
| 16 | 0 | * | 83 | 138 | 0 | * | 0 | * |
| 17 | 0 | * | 231 | 193 | 0 | * | 0 | * |
| 18 | 0 | * | 185 | 236 | 0 | * | 0 | * |
| 19 | 0 | * | 65 | 155 | 0 | * | 0 | * |
| 20 | 0 | * | 7 | 106 | 0 | * | 0 | * |
| 21 | 0 | * | 174 | 37 | 0 | * | 0 | * |
| 22 | 0 | * | 148 | 174 | 0 | * | 0 | * |
| 23 | 0 | * | 202 | 77 | 0 | * | 0 | * |
| 24 | 0 | * | 173 | 11 | 0 | * | 0 | * |
| 25 | 0 | * | 119 | 247 | 0 | * | 0 | * |
| 26 | 0 | * | 11 | 188 | 0 | * | 0 | * |
| 27 | 0 | * | 72 | 208 | 0 | * | 0 | * |
| 28 | 0 | * | 219 | 38 | 0 | * | 0 | * |
| 29 | 0 | * | 169 | 146 | 0 | * | 0 | * |
| 30 | 0 | * | 177 | 115 | 0 | * | 0 | * |
| 31 | 0 | * | 0 | * | 0 | * | 0 | * |
| 32 | 0 | * | 0 | * | 0 | * | 0 | * |

i= 5
d(R)= 1 d(S)= 30 L=-29

1 o log()

0 37 172 (Error-locator polynomial)
1 16 4
2 0 *
3 0 *
4 0 *
5 0 *
6 0 *
7 0 *
8 0 *

| | | |
|----|---|---|
| 9 | 0 | * |
| 10 | 0 | * |
| 11 | 0 | * |
| 12 | 0 | * |
| 13 | 0 | * |
| 14 | 0 | * |
| 15 | 0 | * |
| 16 | 0 | * |
| 17 | 0 | * |
| 18 | 0 | * |
| 19 | 0 | * |
| 20 | 0 | * |
| 21 | 0 | * |
| 22 | 0 | * |
| 23 | 0 | * |
| 24 | 0 | * |
| 25 | 0 | * |
| 26 | 0 | * |
| 27 | 0 | * |
| 28 | 0 | * |
| 29 | 0 | * |
| 30 | 0 | * |
| 31 | 0 | * |
| 32 | 0 | * |

| i | e | log | i | e | log | i | e | log | i | e | log | |
|-----|---|-----|-----|---|-----|-----|---|-----|-----|----|-----|--------------------|
| 0 | 0 | * | 1 | 0 | * | 2 | 0 | * | 3 | 0 | * | (Error pattern) |
| 4 | 0 | * | 5 | 0 | * | 6 | 0 | * | 7 | 30 | 202 | |
| 8 | 0 | * | 9 | 0 | * | 10 | 0 | * | 11 | 0 | * | |
| 12 | 0 | * | 13 | 0 | * | 14 | 0 | * | 15 | 0 | * | |
| 16 | 0 | * | 17 | 0 | * | 18 | 0 | * | 19 | 0 | * | |
| 20 | 0 | * | 21 | 0 | * | 22 | 0 | * | 23 | 0 | * | |
| 24 | 0 | * | 25 | 0 | * | 26 | 0 | * | 27 | 0 | * | |
| 28 | 0 | * | 29 | 0 | * | 30 | 0 | * | 31 | 0 | * | |
| 32 | 0 | * | 33 | 0 | * | 34 | 0 | * | 35 | 0 | * | |
| 36 | 0 | * | 37 | 0 | * | 38 | 0 | * | 39 | 0 | * | |
| 40 | 0 | * | 41 | 0 | * | 42 | 0 | * | 43 | 0 | * | |
| 44 | 0 | * | 45 | 0 | * | 46 | 0 | * | 47 | 0 | * | |
| 48 | 0 | * | 49 | 0 | * | 50 | 0 | * | 51 | 0 | * | |
| 52 | 0 | * | 53 | 0 | * | 54 | 0 | * | 55 | 0 | * | |
| 56 | 0 | * | 57 | 0 | * | 58 | 0 | * | 59 | 0 | * | |
| 60 | 0 | * | 61 | 0 | * | 62 | 0 | * | 63 | 0 | * | |
| 64 | 0 | * | 65 | 0 | * | 66 | 0 | * | 67 | 0 | * | |
| 68 | 0 | * | 69 | 0 | * | 70 | 0 | * | 71 | 0 | * | |
| 72 | 0 | * | 73 | 0 | * | 74 | 0 | * | 75 | 0 | * | |
| 76 | 0 | * | 77 | 0 | * | 78 | 0 | * | 79 | 0 | * | |
| 80 | 0 | * | 81 | 0 | * | 82 | 0 | * | 83 | 0 | * | |
| 84 | 0 | * | 85 | 0 | * | 86 | 0 | * | 87 | 0 | * | |
| 88 | 0 | * | 89 | 0 | * | 90 | 0 | * | 91 | 0 | * | |
| 92 | 0 | * | 93 | 0 | * | 94 | 0 | * | 95 | 0 | * | |
| 96 | 0 | * | 97 | 0 | * | 98 | 0 | * | 99 | 0 | * | |
| 100 | 0 | * | 101 | 0 | * | 102 | 0 | * | 103 | 0 | * | |
| 104 | 0 | * | 105 | 0 | * | 106 | 0 | * | 107 | 0 | * | |
| 108 | 0 | * | 109 | 0 | * | 110 | 0 | * | 111 | 0 | * | |

| | | | | | | | | | | | |
|-----|---|---|-----|---|---|-----|---|---|-----|---|---|
| 112 | 0 | * | 113 | 0 | * | 114 | 0 | * | 115 | 0 | * |
| 116 | 0 | * | 117 | 0 | * | 118 | 0 | * | 119 | 0 | * |
| 120 | 1 | 0 | 121 | 0 | * | 122 | 0 | * | 123 | 0 | * |
| 124 | 0 | * | 125 | 0 | * | 126 | 0 | * | 127 | 0 | * |
| 128 | 0 | * | 129 | 0 | * | 130 | 0 | * | 131 | 0 | * |
| 132 | 0 | * | 133 | 0 | * | 134 | 0 | * | 135 | 0 | * |
| 136 | 0 | * | 137 | 0 | * | 138 | 0 | * | 139 | 0 | * |
| 140 | 0 | * | 141 | 0 | * | 142 | 0 | * | 143 | 0 | * |
| 144 | 0 | * | 145 | 0 | * | 146 | 0 | * | 147 | 0 | * |
| 148 | 0 | * | 149 | 0 | * | 150 | 0 | * | 151 | 0 | * |
| 152 | 0 | * | 153 | 0 | * | 154 | 0 | * | 155 | 0 | * |
| 156 | 0 | * | 157 | 0 | * | 158 | 0 | * | 159 | 0 | * |
| 160 | 0 | * | 161 | 0 | * | 162 | 0 | * | 163 | 0 | * |
| 164 | 0 | * | 165 | 0 | * | 166 | 0 | * | 167 | 0 | * |
| 168 | 0 | * | 169 | 0 | * | 170 | 0 | * | 171 | 0 | * |
| 172 | 0 | * | 173 | 0 | * | 174 | 0 | * | 175 | 0 | * |
| 176 | 0 | * | 177 | 0 | * | 178 | 0 | * | 179 | 0 | * |
| 180 | 0 | * | 181 | 0 | * | 182 | 0 | * | 183 | 0 | * |
| 184 | 0 | * | 185 | 0 | * | 186 | 0 | * | 187 | 0 | * |
| 188 | 0 | * | 189 | 0 | * | 190 | 0 | * | 191 | 0 | * |
| 192 | 0 | * | 193 | 0 | * | 194 | 0 | * | 195 | 0 | * |
| 196 | 0 | * | 197 | 0 | * | 198 | 0 | * | 199 | 0 | * |
| 200 | 0 | * | 201 | 0 | * | 202 | 0 | * | 203 | 0 | * |
| 204 | 0 | * | 205 | 0 | * | 206 | 0 | * | 207 | 0 | * |
| 208 | 0 | * | 209 | 0 | * | 210 | 0 | * | 211 | 0 | * |
| 212 | 0 | * | 213 | 0 | * | 214 | 0 | * | 215 | 0 | * |
| 216 | 0 | * | 217 | 0 | * | 218 | 0 | * | 219 | 0 | * |
| 220 | 0 | * | 221 | 0 | * | 222 | 0 | * | 223 | 0 | * |
| 224 | 0 | * | 225 | 0 | * | 226 | 0 | * | 227 | 0 | * |
| 228 | 0 | * | 229 | 0 | * | 230 | 0 | * | 231 | 0 | * |
| 232 | 0 | * | 233 | 0 | * | 234 | 0 | * | 235 | 0 | * |
| 236 | 0 | * | 237 | 0 | * | 238 | 0 | * | 239 | 0 | * |
| 240 | 0 | * | 241 | 0 | * | 242 | 0 | * | 243 | 0 | * |
| 244 | 0 | * | 245 | 0 | * | 246 | 0 | * | 247 | 0 | * |
| 248 | 0 | * | 249 | 0 | * | 250 | 0 | * | 251 | 0 | * |
| 252 | 0 | * | 253 | 0 | * | 254 | 0 | * | | | |

BLOCK=

0

SYMERR=

0 BITERR=

0

APPENDIX E

Example of Output for Randomly Chosen Codeword

| 1 | u | 1 | u | 1 | u | 1 | u | (Codeword) |
|-----|-----|-----|-----|-----|-----|-----|-----|------------|
| 0 | 2 | 1 | 143 | 2 | 104 | 3 | 10 | |
| 4 | 157 | 5 | 95 | 6 | 32 | 7 | 68 | |
| 8 | 139 | 9 | 200 | 10 | 143 | 11 | 186 | |
| 12 | 130 | 13 | 130 | 14 | 240 | 15 | 26 | |
| 16 | 124 | 17 | 179 | 18 | 133 | 19 | 176 | |
| 20 | 222 | 21 | 164 | 22 | 4 | 23 | 211 | |
| 24 | 42 | 25 | 248 | 26 | 131 | 27 | 219 | |
| 28 | 37 | 29 | 92 | 30 | 227 | 31 | 151 | |
| 32 | 135 | 33 | 187 | 34 | 189 | 35 | 61 | |
| 36 | 36 | 37 | 246 | 38 | 205 | 39 | 227 | |
| 40 | 155 | 41 | 19 | 42 | 223 | 43 | 111 | |
| 44 | 130 | 45 | 245 | 46 | 58 | 47 | 12 | |
| 48 | 13 | 49 | 141 | 50 | 143 | 51 | 31 | |
| 52 | 65 | 53 | 144 | 54 | 96 | 55 | 187 | |
| 56 | 154 | 57 | 172 | 58 | 239 | 59 | 148 | |
| 60 | 59 | 61 | 55 | 62 | 172 | 63 | 113 | |
| 64 | 154 | 65 | 106 | 66 | 26 | 67 | 38 | |
| 68 | 167 | 69 | 14 | 70 | 69 | 71 | 3 | |
| 72 | 128 | 73 | 182 | 74 | 165 | 75 | 201 | |
| 76 | 148 | 77 | 111 | 78 | 142 | 79 | 30 | |
| 80 | 88 | 81 | 246 | 82 | 29 | 83 | 130 | |
| 84 | 107 | 85 | 106 | 86 | 162 | 87 | 191 | |
| 88 | 72 | 89 | 133 | 90 | 141 | 91 | 234 | |
| 92 | 111 | 93 | 71 | 94 | 216 | 95 | 201 | |
| 96 | 17 | 97 | 52 | 98 | 246 | 99 | 192 | |
| 100 | 62 | 101 | 1 | 102 | 62 | 103 | 95 | |
| 104 | 141 | 105 | 203 | 106 | 215 | 107 | 66 | |
| 108 | 79 | 109 | 203 | 110 | 32 | 111 | 138 | |
| 112 | 49 | 113 | 136 | 114 | 165 | 115 | 209 | |
| 116 | 115 | 117 | 141 | 118 | 129 | 119 | 162 | |
| 120 | 135 | 121 | 157 | 122 | 81 | 123 | 100 | |
| 124 | 86 | 125 | 101 | 126 | 158 | 127 | 215 | |
| 128 | 195 | 129 | 180 | 130 | 199 | 131 | 5 | |
| 132 | 253 | 133 | 41 | 134 | 111 | 135 | 177 | |
| 136 | 27 | 137 | 109 | 138 | 107 | 139 | 84 | |
| 140 | 72 | 141 | 226 | 142 | 39 | 143 | 138 | |
| 144 | 112 | 145 | 220 | 146 | 156 | 147 | 9 | |
| 148 | 111 | 149 | 81 | 150 | 177 | 151 | 20 | |
| 152 | 185 | 153 | 14 | 154 | 50 | 155 | 112 | |
| 156 | 135 | 157 | 108 | 158 | 52 | 159 | 216 | |
| 160 | 133 | 161 | 132 | 162 | 2 | 163 | 237 | |
| 164 | 252 | 165 | 224 | 166 | 142 | 167 | 178 | |
| 168 | 126 | 169 | 180 | 170 | 87 | 171 | 121 | |
| 172 | 21 | 173 | 143 | 174 | 217 | 175 | 88 | |
| 176 | 234 | 177 | 142 | 178 | 120 | 179 | 33 | |
| 180 | 117 | 181 | 17 | 182 | 235 | 183 | 211 | |
| 184 | 39 | 185 | 243 | 186 | 39 | 187 | 140 | |
| 188 | 151 | 189 | 54 | 190 | 207 | 191 | 3 | |
| 192 | 45 | 193 | 61 | 194 | 30 | 195 | 116 | |
| 196 | 79 | 197 | 128 | 198 | 79 | 199 | 29 | |
| 200 | 13 | 201 | 189 | 202 | 145 | 203 | 43 | |

| | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 204 | 77 | 205 | 172 | 206 | 108 | 207 | 47 |
| 208 | 118 | 209 | 54 | 210 | 177 | 211 | 22 |
| 212 | 155 | 213 | 40 | 214 | 226 | 215 | 153 |
| 216 | 44 | 217 | 101 | 218 | 37 | 219 | 51 |
| 220 | 28 | 221 | 156 | 222 | 167 | 223 | 175 |
| 224 | 80 | 225 | 108 | 226 | 20 | 227 | 122 |
| 228 | 202 | 229 | 251 | 230 | 31 | 231 | 81 |
| 232 | 29 | 233 | 89 | 234 | 159 | 235 | 134 |
| 236 | 60 | 237 | 217 | 238 | 91 | 239 | 13 |
| 240 | 243 | 241 | 103 | 242 | 83 | 243 | 142 |
| 244 | 162 | 245 | 69 | 246 | 174 | 247 | 177 |
| 248 | 55 | 249 | 20 | 250 | 163 | 251 | 108 |
| 252 | 191 | 253 | 74 | 254 | 141 | | |

TECHNICAL REPORT STANDARD TITLE PAGE

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